Maxwell and the displacement current

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During the course of the present century, history of science has become increasingly more sophisticated and professional. Improved standards have led to new insights into the nature of science and its growth, and to the exposure as myths of many popular accounts of various episodes in the history of science, including those accounts appearing in science textbooks. Thomas Kuhn (1970) has stressed the inadequacy of textbook history in his book *The Structure of Scientific Revolutions*. There he draws an analogy between history of science as it appears in modern textbooks and official history, as approved by the establishment, in Orwell's novel *1984*. In that novel, history was constantly rewritten in such a way that it invariably appeared consistent with the reigning ideology.

Attempts to justify the false history appearing in textbooks typically appeal to the essential pedagogic character of those books, and it is argued that the reconstructed history is justified by its suitability for illuminating and clarifying the structure and workings of the modern theory. Arguments such as these are by no means conclusive. The essential character and the merit of a theory can be much clarified, and frequently can only be fully grasped when the full power of rival theories and rival approaches is understood, and when the reasons for the superiority of the modern theory over its rivals are appreciated. Such understanding can be enhanced by studying the arguments of the opponents of a theory at the time of that theory's origin and initial fight for survival. The shielding of students from the difficulties and possible limitations of a theory brought to light in this way fosters an uncritical attitude.

The main point of this article is to describe a typical example of false textbook history, namely various accounts of the origin of Maxwell's electromagnetic theory, and to attempt to put the record straight. Maxwell's great successes in electromagnetism stemmed from his introduction of a displace-

ment current into the theory. Once the appropriate form of that current had been introduced, dramatic consequences, such as the propagation of electromagnetic effects in time through empty space, and an electromagnetic theory of light followed from it. Most modern accounts of how Maxwell came to introduce the displacement current, especially those in physics textbooks, are very misleading.

Methodology and influence

One of two paths is frequently presented as the one that led Maxwell to his innovation, as Joan Bromberg has indicated (1967–8). Behind the first path lies the symmetry of the basic equations of the theory. It has been suggested, by N R Campbell (1921) for instance, that Maxwell noted that the equations assume an attractive form if a displacement current is introduced. They become symmetrical with respect to an interchange of magnetic and electrical quantities. Alas, Maxwell never wrote the equations in their symmetrical form. The symmetry was hidden from him because of his inclusion of the vector potential, as well as the fields, among the basic equations of his theory.

The second popular version of the path that led Maxwell to his theory construes Maxwell's innovation as following in a natural way from Ampère's work. The relation between electric currents and the magnetic fields they cause, formulated by Ampère, applied only to conduction currents flowing in closed circuits. Several textbook authors, Bleaney and Bleaney (1959) for example, have suggested that a desire to generalize this result so that it would apply to circuits in which the conduction current is not circuital led Maxwell to introduce displacement currents. However, while it is certainly the case that the displacement current in its modern form does render all currents circuital and that the Maxwell equation relating magnetic fields to the currents giving rise to them can be looked on as a generalization of Ampère's result, Maxwell himself did not arrive at his displacement current in this way, as will be clear from what follows. Nietszche once wrote 'unless a man has chaos in him he cannot give birth to a dancing star'. This remark characterizes the way in which Maxwell arrived at the final version of his displacement current more aptly than the neat reconstructions of textbook history.

In his electromagnetic researches Maxwell inherited the general outlook of Faraday in so far as the latter endeavoured to explain electric and magnetic phenomena in terms of processes in the medium surrounding charged bodies, magnets and electric circuits etc, rather than in terms of electrical fluids or particles or the like residing in and flowing through material bodies and acting on each other at a distance, as was the common practice among Continental theorists. Faraday was not able to accommodate his field theory into the newtonian framework that ruled his day. He lacked the mathematical competence. His theory was formulated in a terminology of his own invention, involving concepts such as lines of force, and an electrotonic state. By contrast with Faraday, Maxwell was mathematically competent. Maxwell seems to have regarded his main task to have been the transformation of Faraday's theory into a newtonian mechanical theory of electromagnetism. Whereas the Continental theorists sought a mechanical theory based on a force law between elements of electrical fluid, Maxwell sought a theory based on the mechanical properties of a medium surrounding electrical systems. Eventually this medium was to be identified with the luminiferous ether.

Unified model

A displacement current first made its appearance in Maxwell's long, detailed, technical paper 'On physical lines of force', published in 1861–2 (Niven 1965). I will try to unravel and summarize the main line of Maxwell's not always accurate argument from the details of that paper. In it Maxwell eventually devised a mechanism which was designed to model all electromagnetic phenomena. Maxwell made it quite clear that he was not proposing the details of his model as 'a mode of connexion existing in nature' nor even as one that he would 'willingly assent to as an electrical hypothesis'. He merely hoped to 'clear the way for speculation'. Maxwell's model was intended as an heuristic guide. The analysis which follows casts doubts on the value of the heuristic role played by Maxwell's model.

'Physical lines' was written in four parts. In parts 1 and 2, published in April and May of 1861, Maxwell constructed a model capable of accounting for the formation of magnetic fields by currents, the interaction of magnets and currents with magnetic fields, and electromagnetic induction. He subsequently extended his model, in part 3, to include electrostatics, and thus produced a unified mechanical model of electromagnetic phenomena in general. It is in part 3 that a displacement current first appears and light waves are mentioned. Part 3 was published in January 1862. Part 4 contained an attempted explanation of the Faraday effect and need not concern us here. In order to appreciate the way in which Maxwell's innovations emerge in part 3, it is necessary to understand some details of his model as he developed it in parts 1 and 2.

The fact that magnetic poles attract or repel each other convinced Maxwell, as it had Faraday, that the lines of force emanating from these poles should

represent a state of tension. The effect of a magnetic field on the plane of polarization of light convinced Maxwell that some sort of rotation of an electromagnetic medium must take place about the magnetic lines of force. These two factors together led Maxwell to attempt to represent a magnetic field by a system of vortices, the axes of the vortices corresponding to magnetic lines of force. the dimensions of the vortices were assumed to be small, so that there were many vortices within the volume of a molecule of matter.

Maxwell showed how it was possible to employ vortices to model the forces on magnets and currents. All that was necessary for this purpose was to identify a vector pointing along the axes of the vortices and equal in magnitude to the speed at the circumference of a vortex with the magnetic field H, to identify $1/4\pi$ curl H as the conduction current density and to assume that the density of the medium is proportional to the magnetic permeability μ.

Having constructed this vortex model of the magnetic field, Maxwell wished to specify what caused the medium to move in the way demanded by his model. For this purpose Maxwell introduced numerous minute particles into the picture. Layers of these minute particles separated the medium into spherical cells. The rotation of the material of the cells was to constitute the vortices required by the model, and the particles separating them were to act as idle wheels, rotating in the opposite sense to that of the vortices they separated. The motion of particles from vortex to vortex through the medium was to constitute a current. The tangential action of the particles on the cells in the electromagnetic medium caused the cells to rotate, and so accounted for magnetic fields due to currents. In an insulating medium the motion of the particles was restricted. However, by their own rotation the particles were able to communicate the rotation of the cells from one cell to the next, and so account for the magnetic field in the insulating medium surrounding current-carrying conductors.

In the model, provided the angular velocity of each vortex remained constant, the particles separating the vortices experienced no net force. However, they did experience such a force when a change in the state of rotation of the vortices was transmitted through the medium. It was in this way that the model was able to account for electromagnetic induction.

Maxwell showed how a model of this kind could be adapted to fulfil the aim he had stated at the beginning of the article. The model was capable of accounting for the formation of magnetic fields by closed currents, the interaction of closed currents and magnets, and electromagnetic induction.

Before moving on to part 3, one point should be stressed. In the model as so far developed electric current is identified with the curl of a vector representing the strength of the vortices. Therefore, since it is necessarily the case that a vector which is the curl of a second vector cannot have sources, it is already a necessary consequence of the model of parts 1 and 2 that currents flow in closed circuits.

Electrostatics

In the summer of 1861 Maxwell endeavoured to extend his model to include electrostatics. His stated aim was to explain what was meant by a charged body in terms of his model, and to derive the law of attraction between charged bodies. Maxwell achieved a derivation of Coulomb's law by introducing a displacement and displacement current into the theory. His conception of those quantities differs significantly from the corresponding quantities in the modern theory, and it is only when the differences are appreciated that apparent discrepancies in the signs appearing in some of the equations written by Maxwell can be explained.

The notion that the molecules of a dielectric become polarized in an electric field, one side becoming positively and the other negatively electrified, was inherited by Maxwell from Faraday and Mossotti. Maxwell argued that changes in displacement involve a motion of electricity, and should therefore be treated as currents. It is worth noting that Maxwell's argument for treating changes in displacement as a current is an electrical one, and he subsequently incorporated the conception into his model. The model would have played a more convincing heuristic role if it had been the case that a term emerged from the mechanical model, for mechanical reasons, which could then have been plausibly interpreted as a displacement current.

Maxwell incorporated the notion of dielectric polarization and displacement into his model by assuming that his electromagnetic medium possessed 'elasticity of figure' so that the cells in the medium could be distorted. The polarized molecules of Faraday and Mossotti were thus replaced in Maxwell's treatment by polarized cells in the electromagnetic medium. The distortion of a cell involved a motion of the particles on its surface, and it was this motion of particles that corresponded to displacement current. This current had to be added to the current already introduced earlier in the article, which was the current due to the motion of particles from vortex to vortex through the medium and was equal to $1/4\pi$ curl H. The total current C is thus given by

$$C = \frac{1}{4\pi} \text{curl } H + \frac{dD}{dt}$$

where D is a measure of the distortion of the cells. The total current, then, interpreted in Maxwell's

model as the net motion of particles across unit area in unit time, is the sum of two terms. The first term represents the number of particles per unit area passing per unit time through a medium from vortex to vortex and is circuital. The second term represents the motion of particles arising from the elastic deformation of the ether cells. Only the current represented by the first of these terms, and not the displacement current, causes a rotation of the cells, and hence a magnetic field. Further, the total current is not circuital.

At this stage I would like to suggest a possible reconstruction of the line of thought that led Maxwell to add a displacement current in the way he did. Maxwell considered the particles of his model to 'play the part of electricity' or to 'constitute the matter of electricity'. In view of this it would seem natural to interpret electric charge as an accumulation of the particles. But if all currents flow in closed circuits, as implied by the model described in parts 1 and 2, there never can be an accumulation of particles at any place. Thus a component must be added to the current so that the total current is *no longer* circuital, and this is just what Maxwell did in part 3 of his article. If this reconstruction is correct, then Maxwell's reasoning was just about the opposite of the popular argument of physics textbooks mentioned earlier.

By introducing displacement, in the form of distorted ether cells, into his model, Maxwell was able to identify charge with the accumulation of particles caused by the displacement. More precisely, the charge density e was given by

$$\text{div } \boldsymbol{D} = -e.$$

With the aid of this expression for charge density, Maxwell was able to derive Coulomb's law. But Maxwell realized that he could take his analysis a stage further. By utilizing the connection between charge and displacement and the expression he had derived for Coulomb's law, Maxwell was able to link the coefficient of rigidity of his electromagnetic medium to the ratio of the electromagnetic and electrostatic units of charge, a ratio that had been measured experimentally by Weber and Kaulrausch. With this link established, Maxwell was able to calculate the velocity of transverse waves in the electromagnetic medium. This velocity turned out to be equal to the velocity of light. Maxwell then concluded dramatically, 'we can scarcely avoid the inference that *light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena*'.

Errors and omissions

Described in this fashion, this result of Maxwell's seems an excellent vindication of the heuristic value of Maxwell's model. However, the situation appears in a somewhat different light when it is realized that Maxwell's derivation contains an error, due to a faulty application of elasticity theory. If this error is corrected, we find that Maxwell's model in fact yields a velocity of propagation in the electromagnetic medium which is a factor $2^{1/2}$ smaller than the velocity of light. It was this state of affairs that led Pierre Duhem to accuse Maxwell of adjusting his calculation so that he could arrive at a theory of light which he already had in mind.

Another factor casting doubt on the heuristic value of Maxwell's model takes the force out of Duhem's accusation. For there is no electromagnetic theory of light in 'Physical lines'. The displacement current of that theory did not give rise to a magnetic field, while from the standpoint of modern theory it can be appreciated that it is the magnetic field cause by the displacement current that is essentially responsible for radiation. The only step that Maxwell was able to take towards an electromagnetic theory of light in 'Physical lines' was to demonstrate, albeit erroneously, that the velocity of light in the luminiferous ether is equal to the velocity of transverse vibrations in the electromagnetic ether, an outcome which led Maxwell to identify the two ethers.

Because of the limited and somewhat fortuitous success that can be attributed to Maxwell's model, the way in which Maxwell arrived at the subsequent version of his electromagnetic theory cannot be taken as supporting the view that scientists should be encouraged to construct mechanical models in an attempt to extend their theories, any more than the way in which Kekulé is alleged to have discovered the benzene ring can be taken as suggesting that all scientists should be encouraged to dream.

Maxwell's identification of the electromagnetic and luminiferous ethers in 'Physical lines' stemmed from the mechanical details of his model, a model that Maxwell himself did not regard as a serious conjecture about the actual state of affairs. Maxwell himself saw this as an inadequacy, as we know from his letters, and he immediately set about attempting to free his theory of the mechanical details of his model. The result of his efforts in this direction was his paper 'A dynamical theory of the electromagnetic field', published in 1865. In section 2 of that paper, Maxwell wrote the general equations of his theory. The displacement current that appeared in them was different from the one that appeared in 'Physical lines', and corresponds closely with the modern version, although the change was not explicitly acknowledged by Maxwell. The new version did give rise to a magnetic field and it enabled Maxwell to derive an electromagnetic theory of light.

How did Maxwell hit on the new form for his

displacement current? The best I can suggest by way of an answer is this: Once Maxwell had stumbled on the possible identity of the magnetic and luminiferous ethers in 'Physical lines', his main objective became the deduction of a truly electromagnetic theory of light that was independent of the details of any mechanical model. He then juggled with his theory until he found the form of the displacement current that would enable him to derive a wave equation. It is this displacement current that appears in the general equations of his 'Dynamical theory'.

But this is not the end of the story. Maxwell changed his displacement current in 'Dynamical theory' but there was no corresponding change in his conception of charge. The result was a formal inconsistency in the fundamental equations of his theory. One of Maxwell's equations states that div $\boldsymbol{D} = -e$, while from three of his other equations it is possible to derive div $\boldsymbol{D} = +e$. The source of the contradiction can be explained as follows. In the modern theory, to which Maxwell's equations, with the exception of div $\boldsymbol{D} = -e$, conform, a dielectric in the vicinity of a charged conductor will be polarized. The resulting charge on the surface of the dielectric is quite distinct from, and opposite in sign to, the charge on the adjacent conductor. But Maxwell, following Faraday, wished to identify the charge on the conductor with the polarization charge on the surface of the dielectric. That is why he wrote div $\mathbf{D} = -e$, not realizing that his other equations had already committed him to a charge on a conductor given by div $\mathbf{D} = +e$.

By 1873, when Maxwell wrote his *Treatise on Electricity and Magnetism*, he had corrected this error, although, again, with no acknowledgment. In the *Treatise* he wrote div $\boldsymbol{D} = +e$, thus removing the formal inconsistency. But Maxwell was never able to give an account of electric charge consistent with that sign assignment. His accounts of charge in the *Treatise* and in subsequent writings are confused and obscure, and, according to some authors, blatently inconsistent. 'Never' writes O'Rahilly (1965) 'did a great physicist throw out such a mass of incoherent ideas, calmly pursuing his course with intuitive genius amid a welter of discrepant theories'.

In restrospect it is possible to identify some of the root causes of Maxwell's trouble. One of them was his very natural assumption that all electric currents correspond to a motion of electricity, whatever electricity might be. This is not the case in the modern theory. The displacement current in a vacuum, unlike a conduction current, does not involve a motion of electrons. A second cause of Maxwell's difficulties was the extreme form of his programme. For he wished to eliminate all forms of the electrical fluids and reduce electromagnetic phenomena to the mechanics of an ether. In particular, he wished to explain charge as a discontinuity in a mechanical state of the ether. This led him away from the modern notion of charge, in the form of the electron. While the Faraday–Maxwell approach led to the introduction of fields into electromagnetism, the electron was to evolve from the electrical fluids of the Continental action at a distance theorists. It was H A Lorentz who first reconciled the two schools of thought with his electron theory.

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